

## ELEMENTARY PRESERVICE TEACHERS READING OF EDUCATIVE CURRICULUM: ATTENDING TO EXAMPLES OF STUDENTS' MATHEMATICAL THINKING

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*We present research on how PSTs attended to and made sense of educative features of Standards-based curriculum designed to support teachers' knowledge of students' mathematical thinking. Data analyzed from 47 PSTs' written responses to five mathematics lessons demonstrate PSTs attended to these features roughly 25% of the time, but often made sense of them in an educative manner when doing so.*

Keywords: Teacher Education-Preservice, Curriculum, Elementary School Education

### Introduction

In 1996, Ball and Cohen called for the design of curriculum materials that could “contribute to professional practice” (1996, p. 7) by supporting teacher learning in addition to student learning. In their response to this call, Davis and Krajcik (2005) presented a series of five “high-level guidelines” describing the roles educative curriculum could play in promoting teacher learning. These guidelines posited curriculum materials could be designed to: 1) support teachers’ development of pedagogical content knowledge (as in Shulman, 1986); 2) support development of teachers’ subject matter knowledge; 3) “help teachers consider ways of relating units during the year” (p. 5); 4) “make visible the developers’ pedagogical judgments” (p. 5); and 5) “promote teachers’ pedagogical design capacity” (p. 5) (as in Brown & Edelson, 2003).

Several curriculum developers within elementary mathematics have attended to the suggestions proposed by Ball and Cohen and Davis and Krajcik. *Standards-based curriculum* series such as *Math Trailblazers* (UIC, 2008), *Investigations in Number, Data, and Space* (TERC, 2008) and *Everyday Mathematics* (UCSMP, 2007) are examples of curricula that have been explicit in their intent to be educative for teachers and students alike by including educative features. However, little research has been conducted to examine the ways in which pre-service teachers (PSTs) read and interact with these materials. In this paper, we report findings related to PSTs’ noticing of educative features intended to support the development of PSTs’ knowledge of students’ mathematical thinking.

### Theoretical Frame

Effective mathematics teaching requires teachers to employ a variety of knowledge, skills and dispositions. In her handbook chapter on teacher development, Sowder (2007) suggested one goal of teacher development should be to develop teachers’ understanding of how students think about mathematics. Research suggests that teachers who understand how students think about particular mathematical ideas will be better positioned to recognize, interpret and support these ideas in their instruction. Cognitively Guided Instruction (CGI) research has demonstrated teacher knowledge of student thinking, reasoning and strategies can lead to gains in student achievement (Carpenter & Fennema, 1992; Carpenter, Fennema, Franke et al., 2000). More recently, within the Mathematical Knowledge for Teaching (MKT) framework (Ball, Hill, &

Bass, 2005; Ball, Thames, & Phelps, 2008; Hill, Sleep et al., 2008), Ball and her colleagues highlighted Knowledge of Content and Students, “the amalgamated knowledge teachers possess about how students learn” (Hill, Sleep et al., 2007, p. 133), as an important facet of pedagogical content knowledge. Research has also demonstrated a positive correlation between teachers’ mathematical knowledge for teaching and student achievement (Hill, Rowan, & Ball, 2005). In preparing PSTs, it is paramount mathematics teacher educators address the development of this knowledge base. One way to aid PSTs’ understanding of student thinking is by leveraging the educative features of *Standards*-based curriculum materials.

Remillard (1999; 2000), Collopy (2003), as well as Davis and Krajcik (2005) have posited that curriculum materials can be educative for teachers as well as students. In their first high-level guideline, Davis and Krajcik (2005) suggested educative curriculum materials could support teachers’ knowledge of student thinking in two ways. Educative curriculum could be designed to: 1) help teachers anticipate and interpret how students might respond to instructional tasks; and 2) inform teachers of common student conceptions and how they may be addressed (Davis & Krajcik, 2005). Empson & Junk (2004) found that 13 teachers using *Investigations* developed integrated knowledge (knowledge of concepts, procedures, mathematical practices, and children’s thinking) of multiplication. Curriculum developers have taken up Davis and Krajcik’s (2005) guidelines in a variety of ways -- making different design decisions concerning how to incorporate this information into their materials for teachers. In this study, we were curious to examine how different *Standards*-based curriculum materials responded to and operationalized these particular guidelines, as well as how our PSTs interpreted this information when reading the curriculum materials.

In order to help frame our understanding of PSTs’ reading of curriculum materials, we employ the construct of teacher noticing. Teacher noticing encompasses two main processes – *attending* to particular events in an instructional setting and *making sense* of those events (Sherin, Jacobs, & Philipp, 2011). Although teacher noticing has been mainly used to describe the active process a teacher engages in while in the act of teaching, we view these two aspects as applicable to teachers’ reading of curriculum materials, particularly those materials designed to be educative as they contain myriad visual information for teachers to process and make sense of. In our study we use the teacher noticing framework to see if PSTs are, in fact, attending to these particular educative features and, if so, if they are making sense of them in ways that promote the development of their knowledge of students’ thinking.

## Methods

Data for this study came from 47 PSTs from two different university sites. Each PST was enrolled in an elementary mathematics methods course jointly designed by the three authors and taught by either the first or second authors. One of the big ideas of our methods course is the idea that children’s mathematical understandings emerge from solving problems and teachers can use questioning to scaffold the development of children’s mathematical understanding and sense making. We employ the use of *Standards*-based curriculum materials as a way to develop PSTs’ knowledge of students’ mathematical thinking, which contributes to our big idea. PSTs were asked to read a series of five lessons, selected to represent a variety of topics, grade levels and educative curriculum series (2 lessons from *Investigations*, 2 from *Math Trailblazers*, and 1 from *Everyday Mathematics*). After reading each lesson, PSTs were asked to answer three questions based upon their reading. Here we report on the second of the three questions, which was same in each assignment and was designed to elicit to what and how PSTs were attending in their reading of the lesson materials. The question asked, “How does this lesson plan help you teach

this particular concept? In other words, what information does it give you that is most important or helpful? Please be specific.” The reading of the five lessons and their responses were homework assignments given over the course of the semester. We provided multiple opportunities for PSTs to read and respond across the course. On the day a lesson and response was due, a portion of class was devoted to a discussion of the educative features of the lesson.

### Analysis

Prior to our analysis of the PSTs’ responses, the first two authors jointly identified and labeled the features within each lesson plan (educative as well as non-educative). Next, we individually coded the educative features with one of Davis and Krajcik’s five high-level guidelines (2005). The established labels for the features of each lesson were used to identify which features, both educative and non-educative, PSTs attended to. For this report, we then identified the educative features in each lesson that could support teachers’ understanding of students’ mathematical thinking in some manner.

PSTs’ responses to the second question were first unitized into differing ideas, separating each response by the various features of each lesson to which the PST was attending. These responses were then double-coded by the first two authors. We first used the labels to code each unit for *what* feature of the lesson PSTs were attending to. Inter-rater reliability for this coding was 90% and discrepancies were resolved by discussion.

We also coded each unit according the manner in which PSTs made sense of the identified lesson facet. We developed an initial series of five codes using the literature related to teachers’ use of curriculum materials. After several passes through the data, the codes were refined and operationally defined. The first two authors independently assigned all data units one of five codes (descriptive, interpretive, evaluative, adaptive, and educative) through a process of focused coding (Charmaz, 2004). A *descriptive* response described the lesson feature, with no indication as to why it was important, only that it was included. In an *interpretive* response, PSTs described the feature and interpreted the lesson feature using their own or prior knowledge (not information given in the materials); indicating to us that they did not learn anything new from the lesson materials. An *evaluative* response described the feature and included a value judgment as to whether the PST believed feature would be effective or not, or if the PST “liked” the feature or not. An *adaptive* response described how the PST might modify or adapt the feature using her own knowledge or ideas. Responses coded as *educative* showed evidence of the feature informing the PSTs’ knowledge in some way (as per Davis & Krajcik, 2005), beyond directions of what the teacher should do in teaching the lesson. Inter-rater reliability for this coding was 87%. Discrepancies were resolved by discussion.

For this specific report, we examined PSTs’ data that attended to the educative features identified as supporting teachers’ understanding of students’ mathematical thinking.

### Results

We examined data from 47 PSTs to answer our research question, “How do PSTs read educative features of curriculum materials designed to support teachers’ knowledge of students’ mathematical thinking?” In the following sections we identify and explain these educative features within the five lessons PSTs were assigned to read, which features were attended to by PSTs and the ways in which PSTs made sense of these features.

In our analysis of the curriculum materials, we determined each of the five lessons provided information for teachers that would support their knowledge of students’ mathematical thinking. These supports were presented in various formats across curriculum series, and were often

presented differently within the same lesson. We saw these features as educative, however in our analysis we found PSTs did not always attend to them, and even when doing so, did not always make sense of them in an educative manner.

### **Stickers: A Base Ten Model**

In the 3<sup>rd</sup> grade *Investigations* lesson, “Stickers: A Base Ten Model”, students explore what happens when 10 is added to an existing two-digit number. At one point in the lesson, the teacher is to ask students to count the number of stickers represented on an overhead sheet (4 strips of 10 and 6 single stickers) and to tell how they counted. The materials then present the phrase “Students might say:” (TERC, 2008, p. 28) followed by photos of two individual student faces and possible ways students might count the stickers. Later in the lesson, after the teacher and students have created a chart with various representations of the problems they have completed, the teacher is directed to ask students, “What digit changed every time Pilar bought more strips of 10? How did it change?” (TERC, 2008, p. 30) In this case, there are no examples of specific student talk presented. Rather, the lesson reads,

The students may notice that the digit in the 10s place increases by 1 each time another 10 is added and increases by 2 when two 10s are added. Use the class pocket 100 chart to help students understand that the 1 and 2 represent 10 and 20 being added to the starting number of stickers. (TERC, 2008, p. 30)

These two facets of the lesson were educative in supporting teachers’ understanding of student thinking, albeit in slightly different ways. The example of student talk serves to help teachers anticipate what counting strategies students might use. The second facet informs the teacher that some students will have picked up on the pattern at this point in the lesson and suggests a scaffold to use in addressing those who do not.

Of the 47 PSTs, 7 (14.9%) attended to the “Student might say:” feature in their written responses. All seven who attended to this feature addressed it in an educative manner within their response. Madison’s (all student names are pseudonyms) answer was representative of these responses, “Another helping tip is that it gives me exact questions to ask the students and some sample responses. This will help me to be better prepared and know how to answer students’ questions before they ask them”.

Eleven PSTs (23.4%) attended to the “Student may notice” piece of student thinking; seven PSTs attended to the feature in an educative manner, while four did so interpretively. Ellie attended to the information in an educative manner, as evidenced in her response:

One of the key things for the students to understand place value is for them to be able to see where the changes are. With this chart they are able to see how the number in the 10s place continually changes, but the 6 in the ones place remains constant. This pattern is a great way to explain to them what the 10s place is and that the number is not changing by 1, but rather each change of 1 in that place is actually a change by 10.

Ralph’s answer was one of the responses coded as interpretive, as he interpreted the information on student thinking as something to assess against and teach directly if needed.

[T]he teacher will expressly ask “What digit changed every time...?” and draws the student’s attention to the work they have been doing. (Specifically, the tens digit, reinforcing/teaching the concept of place value.) Some students will most likely already “get it” by that point, but this is some direct teaching that must be done for this concept.

### **Field Day Refreshments**

The 5<sup>th</sup> grade *Investigations* lesson “Field Day Refreshments” asks students to solve a multistep word problem with a variety of correct solutions. The lesson materials support

teachers' ability to anticipate student responses by including two examples of Sample Student Work. The work is presented in student handwriting as if it were authentic. In this lesson, 8 PSTs (17.0%) attended to this support. 5 PSTs responses were similar to in nature to Amanda's response, "It shows examples of possible student strategies for solving the problem" and coded as descriptive. Three responses were coded as educative. These, like Marilyn's response below, recognized the student work as informing their understanding of what students might do in solving the task and how that would help her in the course of teaching the lesson:

The lesson also offers copies of two students' strategies for solving the problems, with their actual work displayed. This allows the teacher to know what to expect even before teaching the lesson so that the teacher can be prepared as to what questions to ask during discussion or what strategies may be used in his/her own classroom.

### The 500 Hats

In "The 500 Hats", a 3<sup>rd</sup> grade lesson from *Math Trailblazers*, students are to solve a variety of three-digit addition and subtraction word problems using their own strategies set within the context of "The 500 Hats of Bartholomew Cubbins" by Dr. Suess. The materials remind teachers to "Emphasize partitioning numbers and using mental strategies, such as partitioning and combining, doubling, using compliments of ten and a hundred, or skip counting" (UIC, 2008, p. 29). We viewed the information on what strategies to emphasize as supporting teachers' knowledge of students' thinking as it was a reminder to look for and encourage students' own strategies. Students' use of their own strategies was reinforced in the lesson materials, "Keep in mind, however, that the goal of this activity is to engage students in using their own strategies to add and subtract numbers rather than to teach them formal processes" (UIC, 2008, p. 29).

The "Emphasize strategies" feature was, by far, the most attended to within the five lessons as 34 of 47 (72.3%) of PSTs attended to this lesson feature. PSTs' responses were coded in each of the five categories (see Table 1) with descriptive and educative occurring the most frequently. In Table 2 we present sample responses for each of the five categories of codes for this feature.

**Table 1: How PSTs Attended to "Emphasize strategies": The 500 Hats (N = 47)**

Feature	Total	Descriptive	Interpretive	Educative	Evaluative	Adaptive
Emphasize strategies	34	13	5	11	4	1

**Table 2: Examples of How PSTs Attended to "Emphasize strategies": The 500 Hats**

Code/PST	Response
Educative Nancy	There was also a small section with strategies to emphasize to students while they are solving problems. I would use this to look for the use of those strategies in students work and ask them to share them with the class.
Descriptive Kara	The lesson also emphasizes which concepts and operations should be applied to the story's mathematical problems (partitioning numbers and using mental strategies such as partitioning and combining, doubling, using complements of ten and one hundred, and skip counting).
Evaluative Mandy	I also like that students are able to use pencil and paper as well as base tens and hundreds to help them organize their numbers. This allows each student to work at their level and needs and leads to many different strategies
Adaptive Kellie	By suggesting various manipulatives that the students can use, like the base-10 blocks, or listing specific strategies to lead students to, the teacher can be more aware in how to phrase questions and responses. With careful planning, the way



	in which the problems are discussed can be directly related to the use of these strategies and get students to think about using them when solving other problems.
Interpretive Cynthia	Students are able to use their mental math skills while solving the addition and subtraction problems of two or three digit numbers. The students are producing their own strategies and figuring out which ones work best for them individually. As a teacher, I would be able to see which types of strategies my students are using and which strategies are working best for them.

### Counting One Hundred Seventy-Two

“Counting One Hundred Seventy-Two” is a 1<sup>st</sup> grade *Math Trailblazers* lesson focused on representing numbers greater than 100 in a variety of ways. We identified four facets of the lesson that could support teachers’ knowledge of student mathematical thinking. The lesson begins with the teacher writing several numbers between 101 and 199 on the board and asking students “What does each number mean?” (UIC, 2008, p. 35). The lesson includes four examples of what students might say about the meaning of the number 125. As students work on drawing representations of 172 beans, the lesson informs teachers as to what they may do, “Students may try to draw 172 beans, while others will show groups of beans in containers, organized in some way” (UIC, 2008, p.35). The “Assessment” section of the lesson asks teachers to use students’ journal entries to assess their strategy for counting and grouping objects. It explicitly reminds teachers, “Some students may count by ones. Even though counting by ones is an inefficient strategy, it works if done carefully” (p. 37). The lesson also lists appropriate strategies for students to use in completing the Math Facts practice sheet.

Each of these features can support teachers’ knowledge of student thinking by: providing sample responses and common student strategies. Further, the lesson materials not only help the teacher anticipate that some students will attempt to draw 172 individual objects, or to count by ones, but also remind them these strategies can work and are acceptable.

As Table 3 demonstrates, 18 PSTs (38.3%) attended to the “Meanings of 125” information within the lesson materials. There were also 18 PSTs (38.3%) who attended to the “Drawing 172” task and the ways in which students might respond. Thirteen PSTs attended to the assessment section of the lesson materials and the student strategies for counting or grouping objects. Finally, 2 PSTs (4.3%) attended to the “Math facts strategies”.

**Table 3: How PSTs Attended to Educative Supports: Counting 172 (N = 47)**

Feature	Total	Descriptive	Interpretive	Educative	Evaluative	Adaptive
Meanings of 125	18	4	5	9	0	0
Drawing 172	18	6	10	1	1	0
Assessment	13	6	3	1	2	1
Math fact strategies	2	1	0	1	0	0

### Formula for Area of a Triangle

The 4<sup>th</sup> grade *Everyday Mathematics* lesson “Formula for Area of a Triangle” is designed to help students generate the formula for the area of a triangle. The main task involves students determining the length of the base and height, and area of a series of given triangles by counting unit squares on a grid. Then students compare the area of the triangles to the area of parallelogram formed by two congruent copies of the given triangles. The lesson begins with a Math Message asking students to “Make a list of everything that you know about triangles”

(UCSMP, 2007, p. 694). In discussing results from this activity, the Math Message Follow Up informs teachers, “The list might include:” and presents a series of six statements regarding triangles. Following the main task of the lesson, students are asked to complete a page of problems. Problem 8 on this page asks students to compare the areas of two shapes, a star and a square. The materials include insight as to how students sometimes respond to this task, “It may surprise students that the star and the square in Problem 8 have the same area. One way to find the area of the star is to think of it as a square with a triangle attached to each of its sides” (UCSMP, 2007, p. 696). The Math Message Follow Up not only provides sample student responses, it simultaneously supports teachers’ content knowledge of triangles, their features and classification. The support for Problem 8 lets teachers know students may struggle with solving the task, as well as making sense of its answer. It also provides content knowledge support by including one way to think about the task.

The “Math Message follow up” was attended to by 5 PSTs (10.6%). Two PSTs’ responses were coded as educative. Cleo wrote, “The lesson also gives specific examples on page 694 of what students might say when asked about the properties of a triangle. That list is a good resource for how you may want to guide the discussion”. Three responses were coded as interpretive, represented here in what Stacey wrote,

The math message follow up shows me that students will have a chance to develop a list of properties of a triangle. This shows that the students know what a triangle is and some rules or facts associated with triangles. They will then use this knowledge to help them conjure a formula for area. Without this knowledge, they wouldn’t have a starting place to come up with a formula to represent area.

Three PSTs (6.4%) attended to the information on “Problem 8”, with two responses coded as educative and one as descriptive. Allie’s response was coded as educative,

I also liked the information what might be surprising for students. (Example: the area of a star in comparison to the smaller square on page 696). Receiving information like that is always helpful to put student knowledge in perspective and help the teacher get on the student’s level of knowledge.

### Discussion and Implications

Within these five lessons, we identified 10 lesson features that could support teachers’ understanding of students’ mathematical thinking. Across the PSTs’ responses to the five lessons, there were a total of 116 instances of PSTs attending to these lesson facets; resulting in PSTs attending to these particular features roughly 25% of the time (116/470; 24.7%). Of the 116 instances of attending, 36 were coded as descriptive (31.0%), 27 (23.3%) as interpretive, 44 (37.9%) as educative, 7 (6.0%) as evaluative, and 2 (1.7%) as adaptive. Our findings indicate PSTs are not attending to these features at a high rate, although when they do, they are most likely to do so in a descriptive or educative manner.

Moving forward, we are interested in examining what factors influence PSTs’ attention when reading educative materials. Does the manner in which the support is presented or what knowledge is supported influence if and how PSTs attend to it? For example, in the Stickers Lesson, we think the “Students might say:” feature is very clear in its intent, even highlighting the information as possible student responses with photos of students. However, more PSTs attended to the “Students might notice” information, even though to us it seemed to be presented in a less obvious manner. Further, all PSTs who attended to the “Students might say” did so in an educative manner, while some PSTs who attended to “Students might notice” did so in an interpretive manner. We believe structure and organization could be factors influencing how

features are attended to, but in light of these results, we posit the content is equally if not more important. That is to say, although both features were intended to be educative for teachers, perhaps our PSTs did not require as much support in understanding how students count as opposed to understanding if students would be able to recognize the pattern in the tens digit without direct instruction, resulting in more PSTs attending to that feature.

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